PROBLEM-SOLVING IN MATHEMATICS EDUCATION

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Abstract. Social constructivism identifies mathematics as the result of posing and solving problems by humans. This study examines various views on problems and problem-solving described in multiple books and documents. This research is a literature study. The study results show that problems and problem-solving have an essential role in mathematics, have been recognised for a long time, and are considered essential skills to master. Experts suggest steps with slightly different details in solving the problem. There are various problem-solving strategies, and teachers can develop these skills in math classes in several ways.

Keywords: problem, problem-solving steps, constructivism

INTRODUCTION

Mathematics develops unexpectedly from problems and conjectures that give rise to debate until a theory is formed (Hersh, 1997). Social constructivism identifies mathematics as a social institution resulting from human problem-posing and solving. Often the techniques designed to solve them represent major advances in mathematics. Problems thus also serve as growth points for mathematics (Ernest, 2004).

Hallet proposed that problems play a vital role in the evaluation of mathematical theories. In the 1960s, the progressive tradition in mathematics became widespread. During this period, a progressive orthodoxy developed, increasing the emphasis on discovery, problem-solving, and children's attitudes toward mathematics. Several of philosophers identified problems and problem-solving as the core of the scientific endeavor (Ernest, 2004). While Laudan (1980) sees science as basically aimed at solving problems.

Mathematical problem-solving and research related to it have been a concern for a long time, as written in the workshop paper entitled Mathematical problem solving (Hatfield, Larry L., Bradbard, 1978), which consists of five articles related to problem-solving, as well as research conducted by Sherman & Fennema (1978), Law (1972), Malin (1979), Evans (1980), and many other studies. Although it has long been introduced, problem-solving is still an ability is a concern and is emphasized in learning and research. Research in recent years related to problem-solving was conducted by Schukajlow et al. (2022), Olugbade et al. (2022), El Bedewy et al. (2021), Zhang et al. (2022), Piñeiro et al. (2022), and many other studies.

Public attention to the importance of problem-solving skills is not only seen in research, but also in the inclusion of problem solving in curriculum documents for a long time until now. In Indonesia, for example, the 1984 curriculum document for senior high school has mentioned problem solving as a General Instructional Objective (Depdikbud, 1985). Even in Indonesia's current curriculum, the curriculum "Merdeka", problem solving is still an important aspect of mathematics. This is evident in the objectives of mathematics as well as the process elements in mathematics (Keputusan Kepala Badan Standar, Kurikulum, dan Asesmen Pendidikan Kemdikbudristek No. 033/H/KR/2022, 2022). Not only the curriculum in Indonesia, curriculum documents from various countries also seem to emphasize problem solving. The curriculum document of the Republic of Estonia states that the general competence developed in students is learning to lean competence, one of which is the ability to use learning outcomes in different situations and to solve problems (National Curriculum for Upper Secondary Schools, 2014). In Ontario, in the Mathematics for Work and Everyday Life course, it is mentioned that students will consolidate their math skills when solving problems and communicating their thinking (The Ontario Curriculum, 2018). The curriculum in Zimbabwe, on the other hand, states that one of the profiles of graduating students is problem-solving skills (Curriculum Framework for Primary and Secondary Education 2015-2022, 2015).

Based on this explanation, problems and problem solving have an important role in mathematics and have been recognized for a long time. School mathematics teaching in various countries is known to have emphasized problem solving. This article examines how problems and problem solving are defined by experts, how are the steps to solving problems, and how problem solving is taught in mathematics classes.

RESEARCH METHOD

This research is a library research. The data sources used in this research are relevant literature such as books, scientific articles, official documents, and official websites related to mathematical problem solving.

RESULT AND ANALYSIS

1.1 Problem and Problem Solving

Problems arise when one has a goal but does not know how it will be achieved. When one cannot move from a particular situation to a desired situation by action alone, one must think of another way (Duncker, 1945). A problem is described as a situation where a person is called upon to perform a task for which he does not have an accessible algorithm for determining the method of solution (Lester, 1978). There is a goal that must be achieved through some action by a person, but the way to achieve it is not immediately apparent (Robertson, 2017). Similarly, Lawson (2003) describes a problem situation as one in which a person does not have an available procedure that will enable him or her to achieve the desired goal. Although effective procedures may eventually be developed or memorized, at the time of the problem, the procedures are not available so the individual must organize about the process of developing procedures or accessing procedures that have already been developed. This process is referred to as problem solving. An individual is said to face a problem when there is an obstacle or gap between where he or she is now and where he or she would like to be (Robertson, 2017). Thus, every problem includes three components: (1) initial state; (2) goal state; and (3) obstacles (Matlin, 2013).

If a problem is considered difficult, then problem solving is defined as overcoming the difficulty (Arıkan & Ünal, 2015). Problem solving refers to situations where prior experience, knowledge, and intuition must be coordinated in an attempt to determine the outcome of a situation where the procedure for determining the outcome is unknown (Lester, 1978). Whereas in NCTM (2000) explains problem solving means that problem solvers are involved in a task whose solution method is not known in advance and to find a solution, problem solvers must utilize their knowledge, and through this process, they often develop new mathematical understanding.

The extent to which a situation will be a problem for an individual depends on the nature of the problem and the resources available to the problem solver. These resources include one's background knowledge and experience, the physical resources available at the time, as well as the nature of the situation in which the task must be accomplished. (Lawson, 2003).

To solve problems, we need to use knowledge such as: (1) mathematical concepts; (2) rules and principles; (3) problem categorization; and (4) domain-relevant semantics. In addition, skills such as: (1) Inferencing; (2) Case-based reasoning; (3) Analysis and synthesis; (4) Progress monitoring; (5) Decision making; (6) Abstraction of the underlying problem structure; and (7) Generalization (the ability to apply what you have learned to new examples) (Robertson, 2017).

Although there are no fixed rules for determining whether a particular problem is interesting, there are some general rules to guide the selection of problems: (1) if written, make sure the problem is easy for students to read; (2) use personal words and terms in the problem statement so that students feel that they are part of the problem; (3) try to use real-world problems because they have high motivational value; (4) encourage students to create their own problems; and (5) avoid putting students in stressful situations, such as urging them to get the right answer in a short time, because this will kill the spirit of working on problems (Lester, 1978).

The types of situations that can be used in problems include real-world applications of mathematics as seen by students, problems related to math lessons according to curriculum standards, math recreation, and problems that involve strategies such as guessing and testing and finding patterns (Lester, 1978). The same view is also expressed by Pellegrino, Chudowsky, & Glaser serta Greeno (dalam Lawson, 2003) that problem solving needs to be represented as a situated activity, an activity that is influenced by the features of the situation in which it is performed, including the cultural context in which the problem situation is situated. Good problems can inspire the exploration of important mathematical ideas, foster perseverance, and reinforce the need to understand and use a variety of mathematical strategies, properties and relationships (NCTM, 2000).

A problem can be categorized (Robertson, 2017):

1. Whether the problem provides the information that needs to be known to solve it or the problem solver has to figure out for themselves what to do;
2. The prior knowledge required to solve it;
3. Whether it is necessary to know a lot about the subject or origin of the problem (such as chemistry, badminton, cooking, etc.) before solving it;
4. The nature of the goal involved;
5. Complexity;
6. Whether or not it is the same as one that has been solved before.

1.2 Problem-Solving Steps

To find a solution to a problem, there are several steps to problem solving according to experts, including:

1. Polya's model

The problem solving steps proposed by Polya in 1945 (Maciejewski, 2018) is a model that is widely used as a reference in solving mathematical problems. Polya suggested four steps to get a solution, including:

a. Understanding the problem. At this stage students identify what is being asked in the problem. It is important to know all the data available in the problem and determine and distinguish the necessary conditions, whether they are sufficient, relevant, redundant, and contradictory or not among the information given.
b. Devising a plan. A well-designed plan will make connections between the data and the unknown. Usually, planning is built on the basis of problem-solving experiences comparable to those that have been done before. Certain techniques and defined outcomes used in previous problem solving can inform the current re-problem statement.
c. Carrying out the plan. It is important to carry out each step of the plan carefully and verify that each step is carried out logically.
d. Looking back. At this stage the problem solver checks the solution obtained from a problem by checking the arguments, ensuring there are no errors in reasoning

2. Johnson

There are three stages in solving the problem proposed by Johnson in 1955 (Lester, 1978), including:

a. Preparation and orientation. At this stage students get an overview of the problem at hand;
b. Production. At this stage students consider alternative approaches to solutions and other possible solutions;
c. Judgment. At this stage students determine the adequacy of the solution and the validity of the approach used to arrive at the solution..

3. D'Zurilla dan Goldfried

D'Zurilla & Goldfried (1968) reviewed various literatures and suggested five phases to train to to solve problems:

a. Orientation;
b. Problem statement and definition;
c. Production of alternatives;
d. Decision-making;
e. verification.

4. Webb

Webb created a problem-solving model that was a synthesis of various models in 1974 (Lester, 1978). Webb created a problem-solving model that was a synthesis of various models in 1974 (Lester, 1978). There are three main stages in solving problems, such as:

a. Preparation. At this stage students define and understand the problem, i.e. understand what is unknown, what is given, and what is the goal;
b. Production. At this stage students look for ways to achieve the goal by recalling principles, facts, and rules from memory, as well as generating new concepts and rules to use in solving the problem, and developing alternative hypotheses and plans that can lead to one or more goals; and
c. Evaluation. At this stage the student checks the subgoals and final solution, and checks the validity of the procedures used in the preparation and working stages.

5. Klausmeir dan Goodwin

The five stages proposed by Klausmeir dan Goodwin in 1966 (Lester, 1978) consist of:

a. Setting goals;
b. Appraising the situation;
c. Striving to achieve the goal;
d. Confirming or rejecting the solution found;
e. Achieving the goal.

6. Indiana University's Problem Solving Working Model

a. Problem awareness. At this stage, a situation is given to the student. Before this situation becomes a problem for the student, the student must realize that there is a difficulty. This means that the situation cannot be solved easily. After awareness, there needs to be a student's willingness to try to solve the problem.
b. Problem comprehension. This stage is the stage after the student realizes the existence of the problem and expresses willingness to solve it, so the task of finding a way out of the problem begins. This stage involves at least two sub-stages, namely translation and internalization. Translation involves the interpretation of the information given in the problem into terms that have meaning to the student. Internalization requires the problem solver to sort out the relevant information and determine how this information relates to each other. This stage results in an internal representation of the problem within the problem solver.
c. Analyzing the goal. This is the stage where students reformulate the problem so that familiar strategies and techniques can be used. For some problems, subgoals may be created in advance to aid problem understanding and procedure development. This stage involves identifying the component parts of a problem.

d. Developing a plan. At this stage the problem solver pays conscious attention to designing a solution plan. This stage involves not only identifying potential strategies, but also sequencing subgoals and determining operations that can be used. Equipping students with multiple strategies can facilitate students' ability to develop a plan. For some students, the hardest part of problem solving lies in knowing what to do first and organizing their ideas.
e. Implementing the plan. At this stage students try to implement the plan that has been prepared. Although the chosen strategy is correct, students are very likely to make calculation errors so that they fail to find the solution correctly.
f. Evaluating procedures and solutions. The role of evaluation in problem solving is not just checking and making sure that answers make sense. It is an ongoing process that begins as soon as the student analyzes the goal and continues until a solution is found. Some questions that can be asked at each stage are:

1) Problem understanding stage: what are the relevant and irrelevant data related to the problem? Do I understand the relationship between the given information? Do I understand the meaning of all the terms?
2) Goal analysis stage: are there sub-goals that I might achieve? Can these subgoals be ranked? Is the ordering of the subgoals correct? Have I correctly identified the conditions that exist in the given problem?
3) Plan development stage: is there more than one way to do this problem? Is there a best way? Have I solved problems like this before? Will the plan lead to a goal or subgoal?
4) Implementing the plan: did I use this strategy correctly? Was the order of the steps in my plan appropriate, or could I have used a different order?
5) Evaluate stage: is the solution generalizable? Does the solution found satisfy all the conditions of the problem? What have I learned that will help me solve other problems?

(Lester, 1978)

Problem-solving steps are certainly not limited to what has been described, but there are many more. Figure 1 below represents a view of the various models. The dashed arrows represent that the problem solver may get stuck or encounter an obstacle which means they may have to go back to the previous phase and start again.

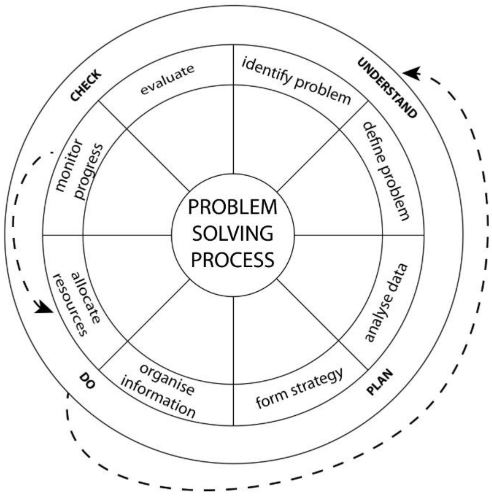

Fig. 1. The problem solving process is seen as a cycle. Rarely are clear linear processes with false starts, dead ends, and so on. (Robertson, 2017).

1.3 Problem Solving Strategy

Mayer (Gick, 1986) explains that problem-solving strategies are techniques that may not guarantee a solution, but serve as a guide in the problem-solving process. There have been many studies on problem-solving strategies that can be used and some of them will be presented in this discussion.

There has been much thought and research into problem-solving strategies. Greenes dan Seymour revealed problem solving strategies that can be taught to students in grades 4-6, including: (1) estimating or guessing, (2) simplifying, (3) doing experiments, (4) making diagrams, (5) making tables, (6) making graphs, (7) writing equations, (8) looking for patterns, (9) making picture charts, (10) dividing space, and (11) deductive logic (Lester, 1978). Gick (1986) explained from various experts about strategies that can be used in solving problems, including (1) problem decomposition, which is breaking the problem into sub-problems; (2) means-ends analysis, which is reducing the difference between the current state and the goal of the problem by applying the right problem-solving operation; (3) working backwards; (4) using analogies by applying similar events; (5) comparing with worked examples or generalization. Meanwhile, Polya revealed several heuristic strategies for solving mathematical problems including (1) drawing, (2) solving simpler analog problems, (3) considering special cases to find general patterns, (4) working backwards, and (5) adopting different points of view (Maciejewski, 2018). Added to NCTM (2000) the strategies proposed by Polya and widely mentioned are: (1) using diagrams, (2) looking for patterns, (3) listing all possibilities, (4) trying special values or cases, (5) working backward, (6) guessing and checking, (7) creating an equivalent problem, and (8) creating a simpler problem.

What is presented by some has similarities, but can be summarized as problem solving strategies including estimating or guessing, simplifying, doing experiments, making diagrams, making tables, making graphs, writing equations, looking for patterns, making picture charts, dividing space, deductive logic, problem decomposition, means-ends analysis, working backwards, using analogies, comparing with worked examples or generalization, listing all possibilities, and trying special values or cases.

Students should know that there are various strategies that can be used in problem solving. When a student uses a strategy, the teacher can inform the student that he/she used a certain strategy and then encourage the student to note it down. This is useful when students solve the next problem. The teacher can also ask other students if they can solve the problem in another way or not. This will help the other students understand what the first student did and show that there is no particular strategy that can be used on all problems encountered. Strategies are learned over time, applied in specific contexts, and become more elaborate and flexible as they are used in increasingly complex problem situations (NCTM, 2000).

1.4 Problem Solving in Learning

Problem solving is seen as an important part of learning mathematics, so educators need to carefully analyze what is involved in the process so that effective learning techniques can be developed. Serious attention should be paid to instructional issues related to problem solving (Lester, 1978). This is because one of the main goals in teaching mathematics is to encourage our students to become good problem solvers (El Sayed, 2002).

Polya (Felmer et al., 2018) reminds us that teachers who have not experienced the "thrill and triumph" of discovery are unlikely to offer problem-solving opportunities to their students in the classroom. For this reason, it is important to develop the professionalism of teachers who teach mathematics. The centrality of problem solving in school mathematics cannot be achieved if teachers are not problem solvers.

There is a lot of research on how problem solving is taught in the classroom. One of the activities that can be done in the classroom related to problem solving is through problem posing. Problem posing is defined as the creation of new problems from certain given events and situations (Arıkan & Ünal, 2015). In more detail, Silver (1994) explains that problem posing refers to the creation of new problems and reformulation of given problems. This posing can occur before, during, or after problem solving. Pellegrino et al (dalam Lawson, 2003) explained that problem posing activities have the potential to move students towards identifying deep structures that characterize more expert performance.

The ideology of progressive educators in mathematics education pays attention to children's feelings, motivations and attitudes (Ernest, 2004). Silver examines this in the practice of problem posing in the classroom. In his article, he explains that problem posing offers a way of connecting math to students' interests. Fifth graders appear highly motivated to pose problems that their classmates think will be interesting or difficult. Students' personal interests are sustained through the process of sharing problems with others (Silver, 1994).

The use of the heuristic approach, according to research, can also improve problem solving skills. The heuristic approach is a mathematical thinking tool to facilitate students in solving mathematical problems (Hoon et al., 2013). Some important heuristics in problem solving (Schoenfeld, 1982) include:

1. At the time of analyzing and understanding the problem: draw diagrams if possible, examine special cases (to exemplify the problem, to explore various possibilities through case restrictions, to find inductive patterns), try to simplify without losing generality.
2. At the time of designing and planning the solution: plan the solution hierarchically, explain what is being done and why, what will be done with the results of these operations.
3. When exploring solutions: consider various equivalent problems (replace conditions with equivalent ones, recombine elements of the problem in different ways, introduce helpful elements, reformulate the problem), consider slight modifications of the original problem (create subgoals and try to solve them, parse the problem and work case by case), consider more extensive modifications of the original problem (examine simpler analogous problems).
4. When verifying: ask whether all the data were used, whether it is a reasonable approximation, can it be obtained in a different way, can it be proved by a special case, can it be reduced to a known result and produce something known?.

The authors argue that the approaches used during learning that aim to improve problem-solving can be used together, given the situation in the math classroom. For example, the use of heuristics can be combined during the application of problem posing. In problem posing, students are required to be able to solve problems that they create themselves. At this stage that heuristics are used. But of course, further research on this is still needed. Based on Polya's description, it is very important to prepare prospective teachers who are good problem solvers so that the centrality of problem-solving in the mathematics classroom can be achieved. University that produces prospective mathematics teachers needs to prepare them well. Of course, attention is not only paid to the problem-solving skills of prospective teachers but also to the skills of teaching problem-solving and attention to students' attitudes towards problem-solving. Research on the learning environment in University that focuses on how to teach problem-solving in the end also needs serious attention.

CONCLUSION

Problems play an important role in the growth of mathematics and as an evaluation of mathematical theory. The ability to solve problems continues to be a concern by many parties and is emphasized in the curriculum of various countries. Although problems are described with different sentences, a common thread can be drawn that a person faces a problem if he is in an initial condition, has a goal, but there are obstacles to achieving that goal. The obstacle is in the form of not having a procedure to achieve the final goal. To solve a problem, the problem solver must use the knowledge they have and use various strategies and go through several stages/steps, which may need to return to the previous stage if the problem solver is stuck. Problem-solving skills can be enhanced by using a variety of instructional tools and teachers need to pay attention to this. The teacher's ability as a problem solver also needs attention so that the centrality of problem solving in the mathematics classroom can be achieved. In addition to the problem-solving skills themselves, it is equally important to pay attention to students' interest, motivation and attitude when solving problems. The presentation in this article still has limitations, it has not thoroughly examined both the theory in mathematics education and the views of psychology and philosophy as well as the results of research that has actually been done a lot.

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